Approximating stiffness-proportional damping with a viscoelastic model in explicit finite element analyses

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Outline

1. Introduction

2. Theory
   - Rayleigh damping
   - Viscoelasticity

3. Validation
   - 1DOF Bar

4. Conclusion
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Construct damping matrix from mass and stiffness matrices:

$$[C] = \alpha[M] + \beta[K]$$  \hspace{1cm} (1)

Using modal analysis generates modal damping expression

$$\zeta_n = \frac{\alpha}{2\omega_n} + \frac{\beta\omega_n}{2}$$  \hspace{1cm} (2)

Abaqus uses a modified Central Difference Method

$$[M]\{\ddot{d}_n\} + [C]\{\dot{d}_{n-1/2}\} + [K]\{d_n\} = \{F^\text{ext}_n\}$$  \hspace{1cm} (3)

which has a stable time increment of \(\Delta t \leq \frac{2}{\omega_{\text{max}}} \left(\sqrt{1 + \zeta^2_{\text{max}}} - \zeta_{\text{max}}\right)\).
Effect of beta-damping on $\Delta t$

Consider a system with minimum natural frequency 1 rad/s and maximum natural frequency 1000 rad/s. The undamped system has a stable time increment of

$$\Delta t \leq \frac{2}{\omega_{\text{max}}} = 2 \times 10^{-3} \text{ s} \quad (4)$$

To get 1% damping on the lowest mode,

$$\beta = \frac{2 \times 0.01}{\omega_{\text{min}}} = 0.02. \quad (5)$$

The modal damping on the highest mode is now $\zeta_{\text{max}} = (0.02 \times 1000)/2 = 10$, which drops the stable time increment to

$$\Delta t \leq \frac{2}{\omega_{\text{max}}} (\sqrt{1 + 10^2} - 10) \approx 9.98 \times 10^{-5} \text{ s} \quad (6)$$
Linear viscoelasticity

Given a dimensionless relaxation function

\[ g(t) = 1 - \sum_{i=1}^{N} \bar{g}_i^P (1 - e^{-t/\tau_i^G}) \]  

solve by numerically approximating Boltzmann convolution

\[ \sigma(t) = \int_0^t G(t - \tau) \dot{\epsilon}(\tau) d\tau \]  

Assuming \( \dot{\epsilon} = \Delta \epsilon / \Delta t \) generates a strain update for each Maxwell element

\[ \Delta \epsilon_i = \frac{\tau_i}{\Delta t} \left( \frac{\Delta t}{\tau_i} + e^{-\Delta t/\tau_i} - 1 \right) \Delta \epsilon + \left( 1 - e^{-\Delta t/\tau_i} \right) (\epsilon_n - \epsilon_{i,n}) \]
Viscoelastic model

$\beta$-damping (1D)

\[ E \]

Standard Linear Solid

\[ E_2 = \gamma E_1 \]
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Case study: 1D bar

Examined the tip displacement of a 1DOF bar when excited by an impulse at the tip. The model was constructed of 100 T3D2 (3D truss) elements.

The force impulse at the tip was characterized by a shifted cosine with a bandwidth designed to excite only the first 10 modes.

Table: System parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>$1 \times 10^{10}$ N/m$^2$</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.3</td>
</tr>
<tr>
<td>$\rho$</td>
<td>1000 kg</td>
</tr>
<tr>
<td>$L$</td>
<td>1 m</td>
</tr>
<tr>
<td>$A$</td>
<td>0.01 m$^2$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$4.026 \times 10^{-6}$</td>
</tr>
</tbody>
</table>
Comparison of results
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<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>0.1</th>
<th>0.25</th>
<th>0.5</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSE</td>
<td>2.54e-009</td>
<td>2.44e-008</td>
<td>8.31e-011</td>
<td>2.16e-011</td>
</tr>
</tbody>
</table>
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Implementing stiffness-proportional damping in an explicit model leads to drastic reductions in the stable time increment, increasing simulation times.

A standard linear solid model can be used to approximate stiffness-proportional damping without the penalty to the stable time increment.
Thank you

Questions?