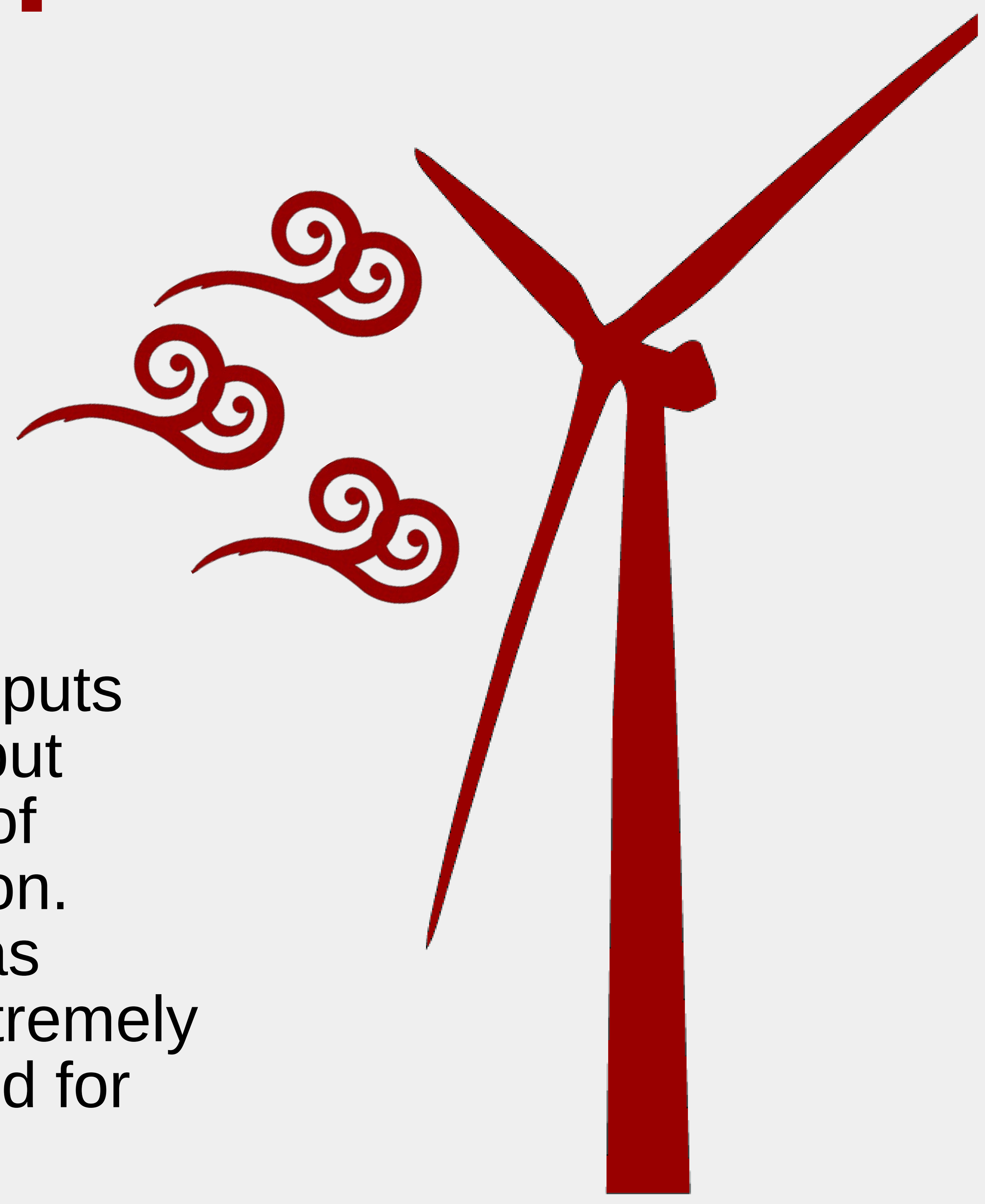
DTU Wind Energy
Department of Wind Energy

Calculating the Sensitivity of Wind Turbine Loads to Wind Inputs using Response Surfaces

Jennifer M. Rinker, Research Assistant



How can we determine which wind inputs affect loads the most?

Sobol Sensitivity Indices (SIs) quantify the degree of influence that model inputs have on a selected model output, and they are useful in determining which input parameters have the most impact on wind turbine loads. The main drawback of Sobol SIs is the large number of model evaluations required for their calculation. This paper presents an approach that uses **polynomial response surfaces** as surrogate models for SI calculations. The proposed methodology performs extremely well, and the calculated SI values demonstrate that Sobol SIs can also be used for **atmospheric model reduction**.

Response Surfaces

Response surfaces are **multidimensional polynomials** that map selected model inputs to a scalar model output:

$$\hat{y}(\boldsymbol{x}) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots$$

Because they are polynomials, querying them only requires matrix arithmetic, making them extremely **fast and efficient surrogate models**.

Fitting Methodology

There are two main steps to fitting a response surface to a set of input/output training data:

1. **Determination of parametric form.** The maximum order of the polynomial terms and what cross-terms to include must be determined. This paper assumes a maximum order for each individual input and eliminates cross-terms from a full factorial if their total power is greater than the maximum individual power.
2. **Calculation of coefficients.** The polynomial coefficients are determined according to least squares:
$$\beta = ([X]^T[X])^{-1}[X]\boldsymbol{y},$$
where $\boldsymbol{y}$ is the vector of measured outputs and $[X]$ is constructed from the measured inputs.

Wind Turbine Model

This paper uses the 5 MW WindPACT reference model, one of a series of models developed in the early 2000s for scaling studies [1] (see Fig. 1). The models are three-bladed, upwind turbines with a variable-speed and collective pitch-to-feather controller. The model was implemented in FAST v7.02 with AeroDyn v13.00, and it uses a Beddoes-Leishman dynamic stall model, equilibrium induction, and standard Prandtl loss models at the hub and tip.

References

[1] Malcolm D and Hansen A 2006 Tech. Rep. NREL/SR-500-32495 National Renewable Energy Laboratory.

[2] Rinker J M, Gavin H P, Clifton A, Veers P S and Kilcher L F 2016 *Boundary-Layer Meteorology*

[3] Kucherenko S, Feil B, Shah N and Mauntz W 2011 *Reliability Engineering & System Safety* 96 440—449.

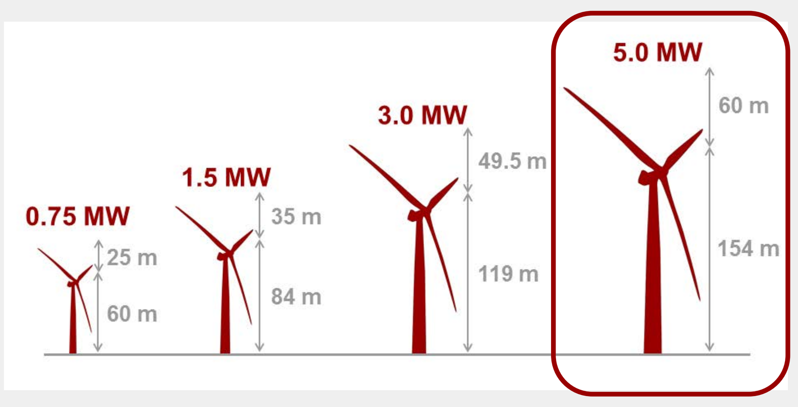


Figure 1: Diagram of WindPACT reference models. The 5 MW model was used in this paper.

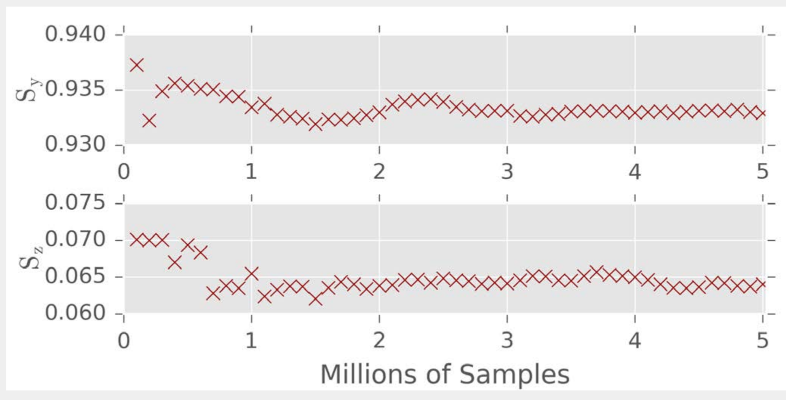


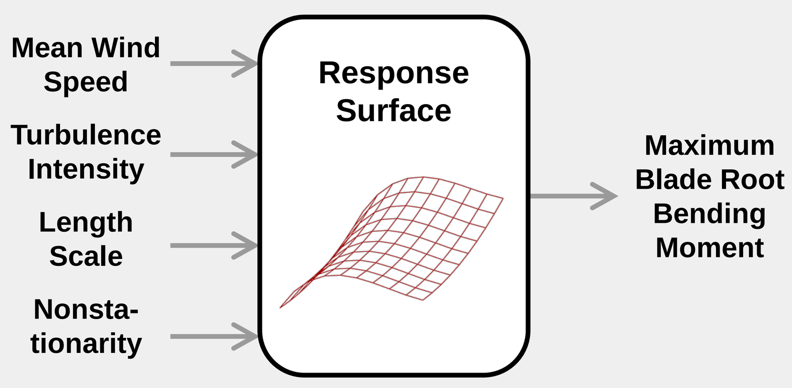
Figure 3: Convergence of Sobol sensitivity indices with respect to the number of Monte Carlo samples.

Developing Surrogate Model

To serve as a surrogate model for Sobol SI calculations, a response surface must be fit to a collection of training data from wind turbine simulations and the quality of the fit must be evaluated.

Model Inputs and Outputs

For this paper, the analyzed model inputs and output are as follows:



The nonstationarity parameter reflects the degree of temporal coherence in the simulated time series [2]; a higher nonstationarity parameter results in higher time-varying turbulence intensity.

Training Data Generation

Thousands of wind turbine simulations were run on a high-performance computing cluster at the National Renewable Energy Laboratory (NREL) with support from Katherine Dykes. The model inputs for the simulations were a hypergrid with the following edge vectors:

$$U_{ref} = [5,7,9,10,10.5,11,11.5,12,13,16,19,22]$$
$$I_{u,ref} = [0.1,0.2,0.3,0.4]$$
$$\log_{10} L_u = [1.5,2.0,2.5,3.0]$$
$$\rho_u = [0.0,0.1,0.2,0.3,0.4]$$

Five redundant simulations were simulated at each grid point to evaluate inherent variance due to random seeds in the stochastic turbulence simulator.

Evaluating Fit of Response Surface

It is important to verify that the response surface adequately models the training data, or the calculated Sobol Sis will be incorrect. Here, the response surface error converges within the five redundant realizations (see Fig. 2), demonstrating the adequacy of the model.

Sampling from Response Surface

Because response surfaces produce deterministic values, it is necessary to add in 10% Gaussian noise to produce stochastic load statistics:

$$Y(\boldsymbol{x}) = \hat{y}(\boldsymbol{x})(1 + 0.1Z), \qquad Z \sim N(0,1)$$

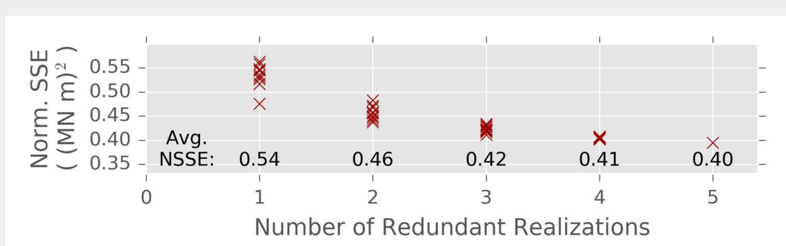


Figure 2: Convergence of response surface error with respect to the number of realizations at each hypergrid point.

Sobol Sensitivities

A Sobol SI is a global sensitivity index that reflects the amount of variance in an output that is caused by a subset of model inputs, y:

$$S_y = \frac{\sigma_y^{\,2}}{\sigma^2}$$

Calculating Sobol SIs requires numerical integration [3], which can be achieved via Monte Carlo integration. For this paper, the wind inputs are grouped into two subsets: $\mathrm{y} = \{U_{ref}, I_{u,ref}\}$ and $\mathrm{z} = \{L_u, \rho_u\}$. Equation 12 from Kucherenko [3] is used to calculate the SIs. The values converge in ≈1.5 million samples to $S_y = 0.933$ and $S_z = 0.064$ (see Fig. 3). This corresponds to the following total sensitivity indices:

$$S_{y,tot} = 0.936 \quad S_{z,tot} = 0.067$$

Because $S_{z,tot} \ll 1$, inputs in z have no significant impact on the model output and can therefore be set to nominal values [3]. Thus, we have determined that the initial atmospheric model with four inputs can be reduced to a model with only two inputs without negligibly affecting the variance of the maximum blade root bending moment.

Significant Conclusions:

- Response surfaces serve well as surrogate models for Sobol SI calculations.
- Of the investigated wind inputs, **only the mean wind speed and turbulence intensity** have a significant impact on the variance of the maximum blade root bending moment.